Gyromagnetic Modes in Waveguide Partially Loaded with Ferrite

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Analysis is made of all the propagating modes of a vanishingly small rectangular waveguide partially filled with transversely magnetized ferrite. Each of these modes is shown to propagate in only one direction and to tend to be lossy. Use of these properties can be made in the design of a novel nonresonance isolator. All but one of the propagating modes vary in amplitude along the dc magnetic field. Yet they can apparently be excited experimentally at a boundary by an incident mode, with none of the modes having any variation along the dc field. Theoretical considerations indicate that finite conductivity in the waveguide walls may be responsible for this coupling.

The unidirectional properties of these modes suggest the possibility of building purely reactive isolators, but these can be shown nonexistent from general energy considerations. Experiments are described that show that nature resolves this "paradox" by absorbing power, even in low-loss ferrite, rather than reflecting it. Some possible explanations of this behavior are set forth.

I. INTRODUCTION

It has been shown that, for certain ranges of transverse magnetic field, there are an infinite number of propagating modes in a waveguide completely filled with ferrite,[1,2,3] no matter how small the guide. These modes we will call gyromagnetic modes, since they have no analog in waveguides filled with isotropic material. For symmetrical structures, the modes of these completely filled waveguides show no nonreciprocal behavior. It is the intention of the present paper to study a similar set of gyromagnetic modes for a waveguide only partially filled with ferrite. Here there are displayed some interesting nonreciprocal effects, which we will describe.

The procedure used will be first to derive explicit expressions for the propagating modes for the partially filled waveguide for a given range

of magnetic fields. We will then sbow bow the nonreciprocal modes obtained can be used in a straightforward fashion to construct a novel isolator. Experimental evidence will be presented that indicates that these higher order modes may be excited from the dominant TE mode, even with boundaries that have no variation in the direction of the applied magnetic field. Some theoretical considerations will indicate that finite conductivity in tbe waveguide walls may be responsible for tbis coupling.

Finally, we will consider the possibility of building purely reaetive isolators. These will be shown to be nonexistent from general energy eonsiderations. But, as Button and Lax[4] have pointed out, tbere are modes which propagate in one direction but are cut off in the reverse, suggesting the possibility of reactive isolation. Some experiments will be described which show that nature resolves this "paradox" by absorbing power rather than by reflecting it. Some possible explanations of this behavior will be set forth.

II. GYROMAGNETIC MODES IN PARTIALLY FILLED RECTANGULAR WAVEGUIDE

We wish to find all of the propagating modes of a rectangular waveguide partially filled with ferrite (Fig. 1). In order to simplify the analysis we will find it convenient to eonsider the waveguide's transverse dimensions to be small compared to a free-space wavelengtb. Altbough this assumption will cut off all of the conventional TE modes, the TE "ferrite dielectric" mode may still propagate, as well as other gyromagnetic modes.

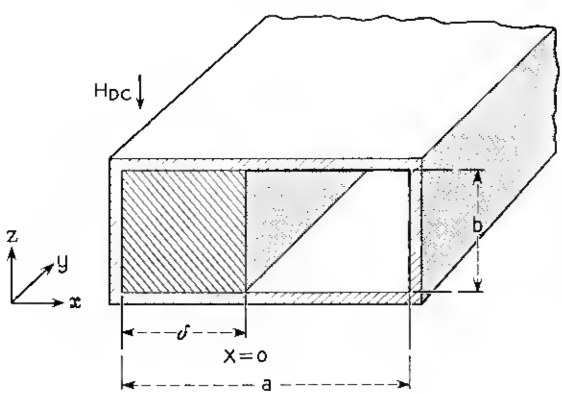

Fig. 1 — Rectangular waveguide partially filled by ferrite slab.

In a loss-free feromagnetic medium, Maxwell's equations are given by

$$\begin{aligned} \text{curl } \mathbf{H} &= i\omega\epsilon\mathbf{E}, \\ \text{curl } \mathbf{E} &= -i\omega\mu_0 T\cdot\mathbf{H}, \end{aligned} \tag{1}$$

where **E** and **H** are the usual field vectors, the time dependence is assumed to be $e^{i\omega t}$ and the tensor T can be written in the Cartesian frame (x,y,z) as

$$T = \begin{pmatrix} \mu & i\kappa & 0 \\ -i\kappa & \mu & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{2}$$

Combining the two equations (1), we obtain the wave equation

$$\nabla \times \nabla \times \mathbf{H} - \omega^2\epsilon\mu_0 T\cdot\mathbf{H} = 0. \tag{3}$$

This has two plane wave solutions, $e^{-i\mathbf{k}_1\cdot\mathbf{R}}$ and $e^{-i\mathbf{k}_2\cdot\mathbf{R}}$. In the limit for which k_x^2, k_y^2 and $k_z^2 \gg \omega^2\mu_0\epsilon$, these solutions are governed[3] by the equations

$$k_{x1}^2 + k_{y1}^2 + \frac{1}{\mu}k_{z1}^2 = 0, \tag{4}$$

$$k_{x2}^2 + k_{y2}^2 + k_{z2}^2 = 0. \tag{5}$$

Equations (4) and (5) determine individual parallel plane-type modes. We will solve these equations for the situation shown in Fig. 1. The fields in the ferrite which satisfy the boundary conditions in the z direction, $E_x = E_y = 0$ at $y = 0$ and $z = b$, can be derived from (1) through (5) as in Ref. 3. For the ferrite mode[1] corresponding to (4):

$$\mathbf{E}_{1f}^{\pm} = F_1^{\pm}\begin{Bmatrix} \left[\left(\dfrac{1-\mu}{\kappa}\right)\cos\varphi_1 \pm i\sin\varphi_1\right]\sin\dfrac{\pi m z}{b} \\ \left[\left(\dfrac{1-\mu}{\kappa}\right)\sin\varphi_1 + i\cos\varphi_1\right]\sin\dfrac{\pi m z}{b} \\ i\mu^{-1/2}\cos\dfrac{\pi m z}{b} \end{Bmatrix} \cdot\exp\left[\frac{\pi m}{b}\mu^{-1/2}(\pm x\sin\varphi_1 + y\cos\varphi_1)\right], \tag{6}$$

$$\mathbf{H}_{1f}^{\pm} = F_1^{\pm} \frac{i\pi m}{\omega\mu_0\mu b}\left(\frac{1-\mu}{\kappa}\right)\begin{pmatrix} \pm \sin\varphi_1 \cos\frac{\pi m z}{b} \\ \cos\varphi_1 \cos\frac{\pi m z}{b} \\ -\mu^{+1/2} \sin\frac{\pi m z}{b} \end{pmatrix} \cdot \exp\left[\frac{m\pi}{b}\mu^{-1/2}(\pm x \sin\varphi_1 + y\cos\varphi_1)\right], \tag{7}$$

where

$$k_{z1} = \frac{m\pi}{b} \qquad (m = 1, 2, 3 \cdots),$$

$$k_{y1} = i\frac{m\pi}{b}\mu^{-1/2}\cos\varphi_1,$$

$$k_{x1} = i\frac{m\pi}{b}\mu^{-1/2}\sin\varphi_1,$$

and the variable φ_1 is introduced for convenience, as in Ref. 3, in place of the propagation constant to be determined by the boundary conditions in the x direction. The superscript + or − on field quantities refers to the two solutions $e^{-ik_x x}$ and $e^{+ik_x x}$ respectively, describing the x variation for the same y variation $e^{-ik_y y}$. The constants F^+ and F^- are the corresponding amplitude constants. Note that $m = 0$ is excluded. The assumption of k_z being very large does not apply to $m = 0$ and hence this will be treated separately. For mode 2 corresponding to (5):

$$\mathbf{E}_{2f}^{\pm} = -F_2^{\pm}\frac{i\pi m}{\omega\epsilon b}\begin{pmatrix} \pm \sin\varphi_2 \sin\frac{\pi m z}{b} \\ \cos\varphi_2 \sin\frac{\pi m z}{b} \\ \cos\frac{\pi m z}{b} \end{pmatrix} \cdot \exp\left[\frac{m\pi}{b}(\pm x\sin\varphi_2 + y\cos\varphi_2)\right], \tag{8}$$

$$\mathbf{H}_{2f}^{\pm} = F_2^{\pm} \begin{bmatrix} \left(\cos\varphi_2 \pm \dfrac{i\kappa}{\mu - 1}\sin\varphi_2\right)\cos\dfrac{\pi m z}{b} \\ \left(\pm\sin\varphi_2 + \dfrac{i\kappa}{\mu - 1}\cos\varphi_2\right)\cos\dfrac{\pi m z}{b} \\ -\dfrac{i\kappa}{\mu - 1}\sin\dfrac{\pi m z}{b} \end{bmatrix} \cdot\exp\left[\frac{m\pi}{b}(\pm x\sin\varphi_2 + y\cos\varphi_2)\right], \tag{9}$$

where

$$k_{z2} = \frac{m\pi}{b},$$

$$k_{y2} = i\frac{m\pi}{b}\cos\varphi_2,$$

$$k_{x2} = i\frac{m\pi}{b}\sin\varphi_2,$$

and φ_2 is the convenient dependent variable for mode 2. Note that, in order for modes 1 and 2 to have the same y variation,

$$\cos\varphi_2 = \mu^{-1/2}\cos\varphi_1 . \tag{10}$$

For the fields in the air region, the two independent plane-wave solutions are both governed by the same equation, which, in the small waveguide approximation, is also the same as the ferrite mode 2 given by (5). For convenience in satisfying the air-ferrite interface boundary conditions, we will not use the usual resolution of these two modes into transverse electric and transverse magnetic. Instead, we will choose one mode so that its tangential electric field can be made continuous with ferrite mode 1 across the interface and the second made so that its tangential magnetic field can be made continuous with ferrite mode 2 across the interface. Thus, for air mode 1 we obtain

$$\mathbf{E}_{1a}^{\pm} = A_1^{\pm} \begin{Bmatrix} \left[\mu^{-1/2}\dfrac{\sin\varphi_1}{\sin\varphi_2}\left(-\dfrac{1-\mu}{\kappa}\cos\varphi_1 \pm i\sin\varphi_2\right)\right]\sin\dfrac{\pi m z}{b} \\ \left(\mp\dfrac{1-\mu}{\kappa}\sin\varphi_1 + i\cos\varphi_2\right)\sin\dfrac{\pi m z}{b} \\ \mu i^{-1/2}\cos\dfrac{\pi m z}{b} \end{Bmatrix} \cdot\exp\left[\frac{\pi m}{b}(\pm x\sin\varphi_2 + y\cos\varphi_2)\right], \tag{11}$$

$$\mathbf{H}_{1a}^{\pm} = A_1^{\pm} \frac{i\pi m}{\omega\mu_0\mu b}\left(\frac{1-\mu}{\kappa}\right)\begin{pmatrix} \cos\dfrac{\pi m z}{b} \\ \pm \dfrac{\cos\varphi_2}{\sin\varphi_2}\cos\dfrac{\pi m z}{b} \\ \mp \dfrac{1}{\sin\varphi_2}\sin\dfrac{\pi m z}{b} \end{pmatrix} \cdot\exp\left[\frac{\pi m}{b}(\pm x\sin\varphi_2 + y\cos\varphi_2)\right] \tag{12}$$

where we have used div $\mathbf{E} = 0$ and div $\mathbf{H} = 0$ to evaluate E_x and H_x and (10) is employed to simplify the expression. For air mode 2

$$\mathbf{E}_{2a}^{\pm} = -A_2^{\pm}\frac{i\pi m}{\omega\epsilon_0 b}\begin{pmatrix} -\sin\varphi_2\sin\dfrac{\pi m z}{b} \\ \mp\cos\varphi_2\sin\dfrac{\pi m z}{b} \\ \mp\cos\dfrac{\pi m z}{b} \end{pmatrix} \cdot\exp\left[\frac{\pi m}{b}(\pm x\sin\varphi_2 + y\cos\varphi_2)\right], \tag{13}$$

$$\mathbf{H}_{2a}^{\pm} = A_2^{\pm}\begin{bmatrix} \left(\mp\cos\varphi_2 + \dfrac{i\kappa}{\mu-1}\sin\varphi_2\right)\cos\dfrac{\pi m z}{b} \\ \left(\sin\varphi_2 \pm \dfrac{i\kappa}{\mu-1}\cos\varphi_2\right)\cos\dfrac{\pi m z}{b} \\ \mp\dfrac{i\kappa}{\mu-1}\sin\dfrac{\pi m z}{b} \end{bmatrix} \cdot\exp\left[\frac{\pi m}{b}(\pm x\sin\varphi_2 + y\cos\varphi_2)\right]. \tag{14}$$

It is easy to verify that these modes satisfy the Maxwell equations in the air region.

The boundary conditions in the x direction now require E_y and E_z to vanish at both metal walls and E_y, E_z, H_y and H_z to be continuous across the ferrite-air interface. This will give us eight linear homogeneous equations to determine the eight unknown constants $A_{1,2}^{\pm}$ and $F_{1,2}^{\pm}$, leading to the secular equation determining the propagation constant.

We wish to concentrate only on those modes that are propagating, i.e., those for which $\cos\varphi_2$ is imaginary. We will also restrict our attention to those modes for which $\mu > 0$, so that $\cos\varphi_1$ will also be imaginary.

This will cause real values for both $\sin\varphi_1$ and $\sin\varphi_2$, so that all the modes will have real exponential decay in the x direction. If we further assume that the waveguide width, a, is much larger than the height, b, we need only consider those modes which decay away from the boundaries.

Thus, at the metal-ferrite wall we need only consider $\mathbf{E}_{1f}^-$ and $\mathbf{E}_{2f}^-$ (taking the sign of $\sin\varphi_1$ and $\sin\varphi_2$ as positive). This leads to exactly the same mode found in Ref. 3 for a completely filled waveguide,

$$\cot\varphi_1 = -i\frac{\mu}{\kappa}. \tag{15}$$

With the use of (9a) and (10), (15) can be solved for k_y:

$$\text{FM:} \qquad k_y = \frac{1}{\kappa}\frac{m\pi}{b}\sqrt{\frac{\mu\kappa^2}{\kappa^2 - \mu^2}}. \tag{15a}$$

We can call this the ferrite-metal (FM) mode since it has a maximum amplitude near the ferrite-metal wall. Notice that there is a solution for this partially filled waveguide only for one direction of propagation for a given value of κ.

At the air-metal wall we need consider only $\mathbf{E}_{1a}^+$ and $\mathbf{E}_{2a}^+$. Since these have exactly the same x dependence there is no nontrivial propagating solution for this case.

At the ferrite-air (FA) interface we need consider only the plus modes in the ferrite and the minus modes in the air. The requirements of continuous E_y, E_z, H_y and H_z lead to the relations

$$\begin{aligned}
&(F_1^+ + A_1^-)\left(\frac{1-\mu}{\kappa}\sin\varphi_1 + i\cos\varphi_1\right) \\
&\qquad\qquad - \left(\frac{F_2^+}{\epsilon} - \frac{A_2^-}{\epsilon_0}\right)\frac{i\pi m}{\omega b}\cos\varphi_2 = 0, \\
&(F_1^+ + A_1^-)(i\mu^{-1/2}) \\
&\qquad\qquad - \left(\frac{F_2^+}{\epsilon} - \frac{A_2^-}{\epsilon_0}\right)\frac{i\pi m}{\omega b} = 0, \\
&\frac{i\pi m}{\omega\mu_0\mu b}\frac{1-\mu}{\kappa}\left(\cos\varphi_1 F_1^+ + \frac{\cos\varphi_2}{\sin\varphi_2}A_1^-\right) \\
&\qquad\qquad + (F_2^+ + A_2^-)\left(-\sin\varphi_2 + \frac{i\kappa}{\mu - 1}\cos\varphi_2\right) = 0, \\
&\frac{i\pi m}{\omega\mu_0\mu b}\frac{1-\mu}{\kappa}\left(-\mu^{-1/2}F_1^+ - \frac{1}{\sin\varphi_2}A_1^-\right) \\
&\qquad\qquad + (F_2^+ + A_2^-)\left(-\frac{i\kappa}{\mu - 1}\right) = 0.
\end{aligned} \tag{16}$$

For solution, the determinant of the coefficients multiplying F_1^+, A_1^-, F_2^+ and A_2^- must vanish. The factoring of this determinant leads to two values of φ_1 :

$$\tan \varphi_1 = -i\frac{\kappa}{\mu}, \tag{17}$$

$$\tan \varphi_2 + \mu \tan \varphi_1 = -i\kappa. \tag{18}$$

These can be solved for the propagation constant with the aid of (9a) and (10):

$$\text{FAI:} \qquad k_y = -\frac{1}{\kappa}\frac{m\pi}{b}\sqrt{\frac{\mu\kappa^2}{\kappa^2 - \mu^2}}, \tag{17a}$$

$$\text{FAII:} \qquad \sqrt{\left(\frac{m\pi}{b}\right)^2 + k_y^2} + \mu\sqrt{\frac{1}{\mu}\left(\frac{m\pi}{b}\right)^2 + k_y^2} = -\kappa k_y . \tag{18a}$$

We will call these the ferrite-air modes (FAI and FAII), since the fields fall off exponentially from the ferrite-air interface.

The modes FM, FAI and FAII represent all the propagating modes for $\mu > 0$ except for the case $m = 0$, for which the approximations used above are not valid. However, $m = 0$ represents a TE mode such as was treated by Button and Lax.[4] In the limit of small waveguide $[b^2 \ll 1/(\omega^2\mu_0\epsilon)]$ the only TE mode that is not cut off is the "ferrite dielectric" mode. Its propagation constant is given by

$$(\mu^2 - \kappa^2)k_a \coth k_a(a - \delta) = \kappa k_y - \mu k_m \coth k_m\delta, \tag{19}$$

where

$$k_a = k_y\sqrt{1 - \frac{\omega^2\mu_0\epsilon}{k_y^2}}, \tag{20}$$

and

$$k_m = k_y\sqrt{1 - \frac{\omega^2\mu_0\epsilon}{k_y^2}\frac{\mu^2 - \kappa^2}{\mu}}, \tag{21}$$

and where δ is the ferrite thickness and a the guidewidth.

A sketch of the propagation constant as a function of magnetic field is shown in Fig. 2 for the various propagation modes. It can be shown from (15a), (17a) and (18a) that the FM mode propagates in the plus y direction between $\mu = \kappa$ and $\mu = 0$, that the FAI mode propagates in the minus y direction between $\mu = \kappa$ and $\mu = 0$ and that the FAII modes propagate in the minus y direction between $\mu = \kappa - 1$ and $\mu = 0$. The FD mode has more complex behavior. For small values of $(a - \delta)/\delta$

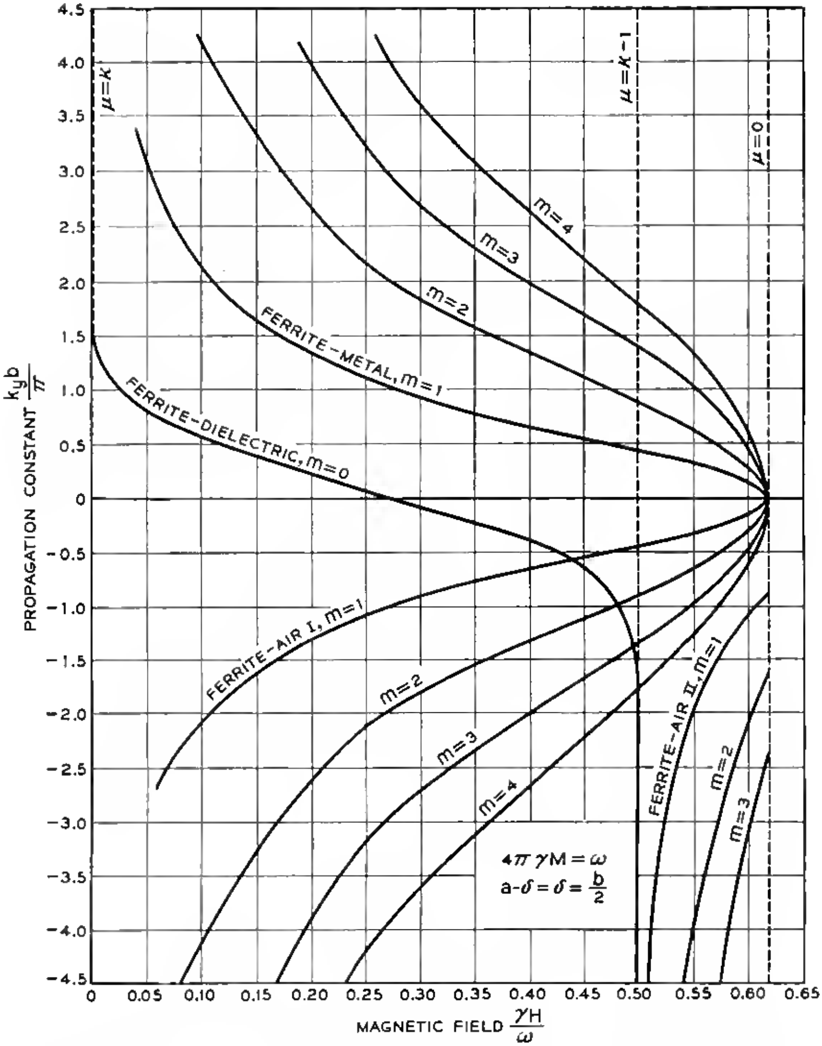

Fig. 2 — Typical mode spectrum of partially filled small guide as a function of magnetic field.

[less than $1 + \omega/(\gamma\pi M)$ for material obeying Polder's relations,[5]], the FD mode has a group velocity in the $+y$ direction between $\mu = \kappa$ and $\mu = \kappa - 1$, but the propagation constant is either plus or minus depending on whether

$$\left(\frac{a - \delta}{\delta} - \frac{\kappa^2 - \mu^2}{\mu}\right)$$

is positive or negative. For large $(a - \delta)/\delta$ [greater than $1 + \omega/(\gamma\pi M)$], the FD mode has both group velocity and phase velocity positive between $\mu = \kappa$ and $\mu = \kappa - 1$, but it is double-valued between $\mu = \kappa - 1$ and $\mu = 0$, having a positive group velocity at whatever magnetic fields it has a negative group velocity.

These gyromagnetic modes tend to be lossy, particularly for the higher orders. This can be demonstrated by allowing μ and κ to be slightly complex: $\mu = \mu' - j\mu''$, $\kappa = \kappa' - j\kappa''$. Then the expressions for the propagation constant can be expanded to give

$$k_y(\mu, \kappa) = k_y(\mu', \kappa') - j\frac{\partial k_y}{\partial \mu}\mu'' - j\frac{\partial k_y}{\partial \kappa}\kappa''. \tag{22}$$

In all the expressions for k_y, FAI, FAII, and FM, $\partial k_y/\partial\mu$ and $\partial k_y/\partial\kappa$ can be seen to be proportional to m. That is, the attenuation increases linearly with the order number, m.

III. GYROMAGNETIC MODE STRIP LINE ISOLATOR

We can use these gyromagnetic modes to make a novel isolator. As a design objective we will try to excite a high-order gyromagnetic mode for one direction of propagation, thus obtaining loss, but will try not to excite any in the opposite direction. To do this we will use a strip line TEM mode incident on a ferrite section, as shown in Fig. 3. The TEM mode has magnetic field components that are symmetric in the z direction and will not couple to the TE mode of the ferrite. However, they are appropriate to couple to the gyromagnetic modes.

To get maximum coupling we need spatial harmonics of the TEM mode to have appreciable amplitudes at the gyromagnetic propagation constant. One way to accomplish this is to break up the ferrite along the y direction.

Now, since the fields fall off in the x direction away from the strip line, we would expect the ferrite-air modes to be excited more than the ferrite-metal modes. Since the FA modes exist for only one direction of propagation and the FM modes for the other, we should get appreciable

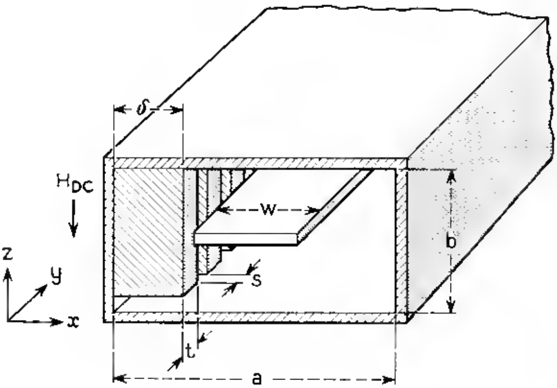

Fig. 3 — Strip line isolator employing periodically broken ferrite slab.

attenuation for the direction of propagation of the FA modes, but little attenuation in the reverse direction.

A physical embodiment of this idea has been built, and a plot of the forward and reverse loss as a function of frequency is shown in Fig. 4. Ratios of reverse to forward loss of greater than 10 can be obtained over a 30 per cent bandwidth. It should be emphasized that this is not a resonance-type isolator. The absorption does not depend on circular polarization, nor does it occur at the ferromagnetic resonance (approxi-

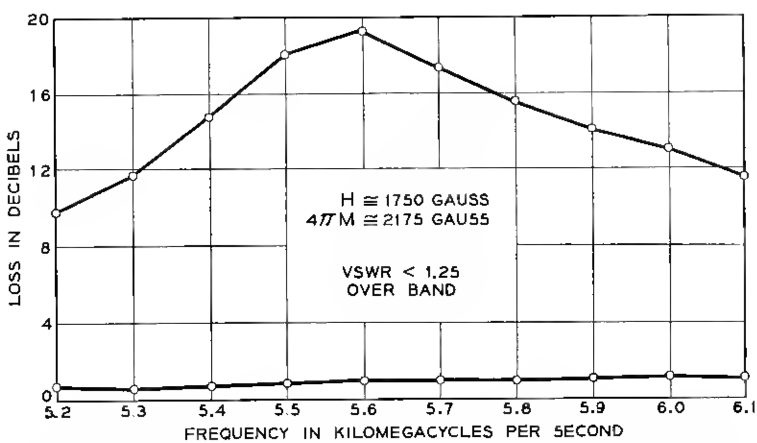

Fig. 4 — Typical characteristic of the gyromagnetic mode strip line isolator of Fig. 3: a = 0.900 inch; b = 0.400 inch; δ = 0.170 inch; S = 0.155 inch; W = 0.588 inch; t = 0.155 inch.

mately 4.4 kmc for the case indicated). On the other hand, the maximum absorption does occur between the frequencies for which $\mu = 0$ (6.2 kmc) and $\mu = \kappa - 1$ (4.5 kmc), as would be expected for coupling to the FAI mode. The small forward loss is believed mainly due to the excitation of the FM mode.

IV. EXCITATION OF GYROMAGNETIC MODES FROM A UNIFORM* TE MODE THROUGH WALL LOSS

We have been surprised to discover experimentally that uniform* TE modes can couple to modes with different symmetry, even when the boundary is uniform.* For instance, let the dominant TE mode of a rectangular guide be allowed to impinge on another rectangular guide containing a slab of ferrite as shown in Fig. 5. The slab completely fills the waveguide in the z direction and ends abruptly on an xz plane. Thus, neither the original mode nor the boundary has any quantity which varies along the magnetic field (z direction). We then insert a probe in the ferrite to probe for nonuniform modes. This probe consists of a metallic plate inserted in the middle of the ferrite slab in the xz plane with metal leads running out in the x direction. Energy will be coupled to this probe only if the average of H_z along the z direction is nonvanishing or if a component E_x appears at $z = b/2$; i.e., only if nonuniform field components appear.

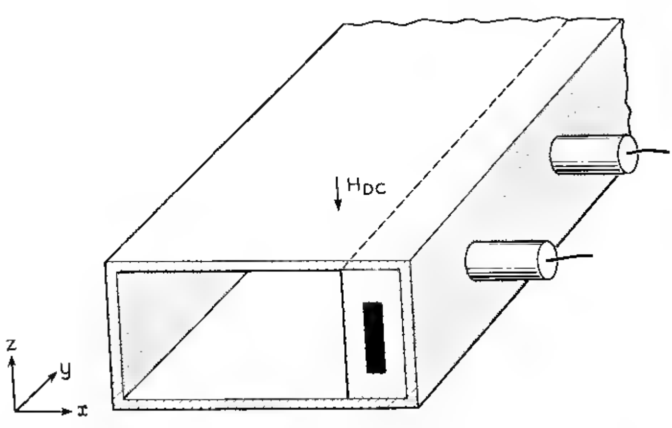

Fig. 5 — Ferrite slab geometry with embedded strip line terminating in coaxial lines.

* By "uniform" we mean to indicate that there are no variations parallel to the applied field.

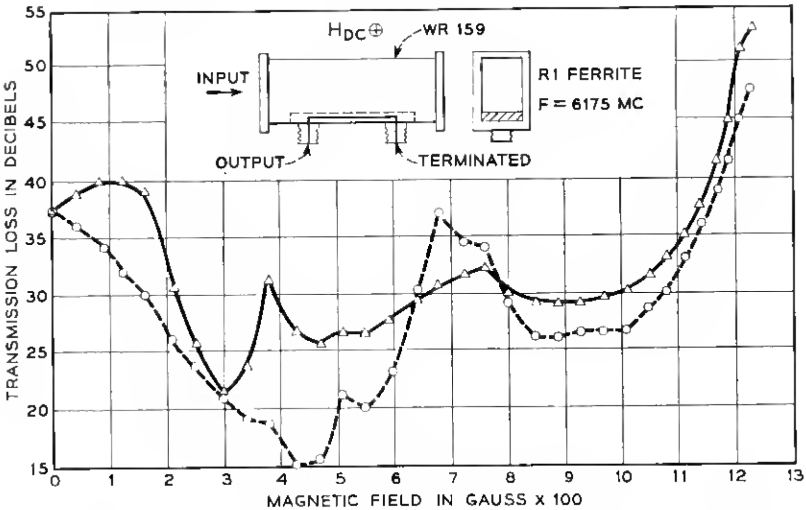

Fig. 6 — Power transmitted to coaxial line with TE incidence on embedded strip line geometry.

In Fig. 6 the fraction of the power coupled to the probe is plotted as a function of magnetic field. In the abseuce of the field, this fraction is below −37 db, the residual presumably being due to small errors in alignment. However, for a particular applied magnetic field this fraction can increase to −15 db, an increase of 22 db. The fact that there are irregularities in the coupling as a function of field suggests an interference between some of the excited modes.

A similar experiment was performed with a full height slab in the center of a square waveguide, away from both side walls (Fig. 7). A dominant mode of a rectangular guide is made incident on this square guide, exciting one polarization. The transverse electric field is probed by examining the transmitted power into a rectangular waveguide at right angles to the first one. Again, if only uniform modes were excited, there should be no transmission. Yet, as shown in Fig. 8, the transmission increases from −47 db at $H = 0$ to as high as −16 db with an applied magnetic field, an increase of 31 db. That this transmission was not Faraday rotation in a small axial magnetic field component could be assured by observing the transmission to be relatively insensitive to a slight tilting of the magnetic field with respect to the waveguide. Both of these experiments indicate the possibility of coupling appreciable power into nonuniform modes.

The only mechanism we have been able to discover which leads to a

compatible model for such a coupling has been the finite conductivity of the walls. Let us consider then what would happen to a TEM wave that was started through a two-dimensional ferrite media contained between metal plates of finite conductivity (Fig. 9). The TEM mode has initially the components E_z and H_x, whose amplitudes are independent of z. As the mode moves through the medium, a magnetic field is developed at right angles to M and H_x; i.e., a field H_y is developed. This field H_y induces currents, $\mathbf{n} \times \mathbf{y}_0 H_y$, in the metal walls. If the walls have a finite conductivity, σ, an electric field will therefore appear equal to

$$\frac{1}{\sigma} \mathbf{n} \times \mathbf{y}_0 H_y .$$

Since at the top face this is opposite in direction from the bottom, the induced electric field in the ferrite medium, E_x, has a z variation which is antisymmetric about the middle of the waveguide, as indicated in Fig. 9.

As the wall conductivity is made ever larger, this antisymmetric component tends to disappear. The limiting processes as this field disappears are, however, very unclear. For instance, we have considered (in Appendix A) the infinite spectrum of modes in the ferrite medium that would be excited by an incident TEM mode perturbed by the finite conductivity of the walls. Under the simple perturbation assumed, we find an unlimitedly large amount of scattered energy is predicted. Since this is impossible, we conclude that the simple perturbation picture is

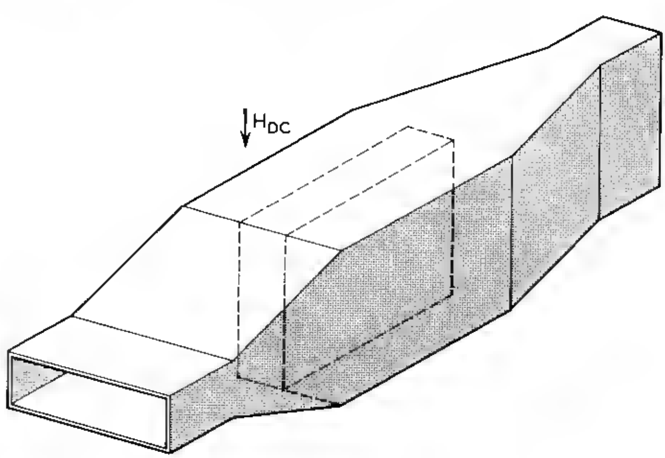

Fig. 7 — Square guide with uniform ferrite slab terminating in orthogonal output.

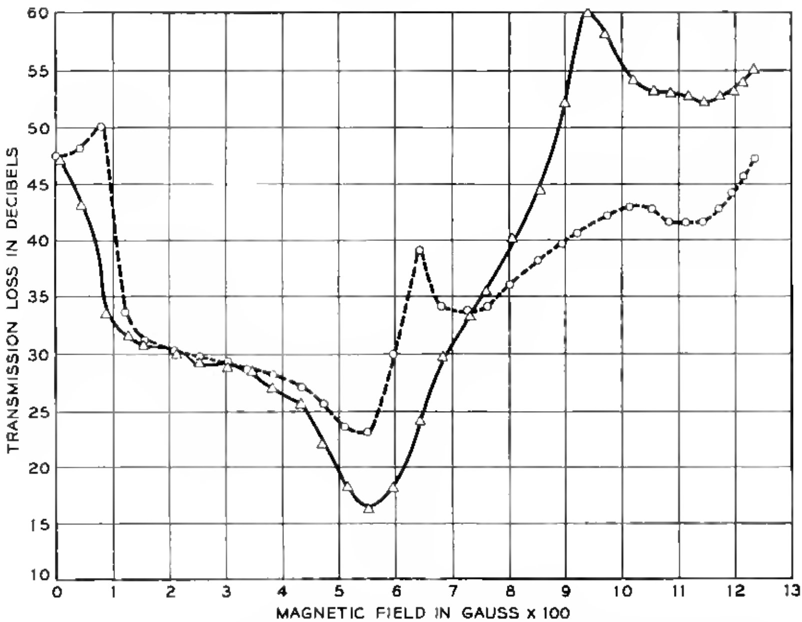

Fig. 8 — Transmission characteristic to orthogonal output by means of a uniform ferrite slab. Solid curve is forward direction; dashed curve is reverse.

incorrect. This leads to the suggestion that modes with variations along the magnetic field are excited at the boundary even in the limit of infinite wall conductivity.

One might wonder why this process of assuming a finite wall resistivity yields a coupling in the limiting process, whereas a starting assumption

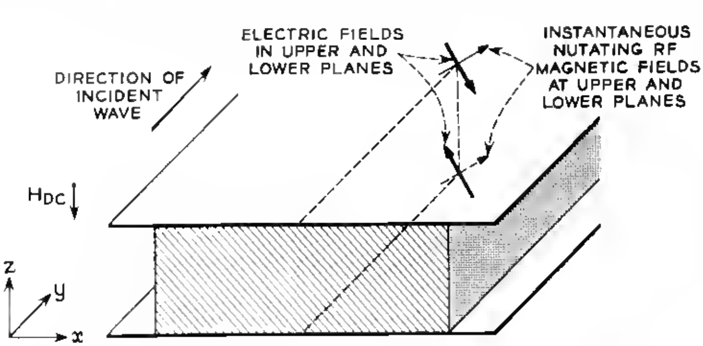

Fig. 9 — Transversely magnetized ferrite slab bounded by finitely conducting parallel planes.

of a loss-free medium does not. It must be remembered that uniqueness of the field representation is obtained only by recognizing the existence of loss terms, however small. Therefore, any reactive physical system has meaning only through this limiting process. Some classical "paradoxes" have owed their existence to the failure to recognize this fact. The above discrepancy in results arrived at by the two processes should therefore not be disturbing.

Thus, although we have not been able to show rigorously that these gyromagnetic modes must be excited from an incident uniform mode in the limit of resistanceless walls, we have demonstrated their excitation experimentally and have produced a plausible explanation of how this might be possible.

V. GENERAL THEOREM ON THE NONEXISTENCE OF PURELY REACTIVE ISOLATORS

We can see from the analyses of Section II and have shown in Fig. 2 that, for $\kappa > \mu > \kappa - 1$, the situation described by Button and Lax[4] is manifested for the ferrite dielectric (FD) mode: it propagates in only one direction. It should be noticed that the same thing is true for any one of the other gyromagnetic modes. This suggests that a lossless device could be made which would be perfectly transmitting in one direction (assuming one could match into one mode, e.g., the ferrite dielectric mode), but be nontransmitting in the opposite direction. However, we can show that such a device is impossible.

Let us consider two reference planes in a waveguide that are so far removed on either side of an arbitrarily loaded section that all modes of the waveguide except a dominant one are vanishingly small. We may then set up a scattering matrix between these reference planes, which we designate 1 and 2, to relate the incident waves, u, and reflected waves, v, at each of these points:

$$\begin{aligned} v_1 &= s_{11}u_1 + s_{12}u_2 , \\ v_2 &= s_{21}u_1 + s_{22}u_2 , \end{aligned} \tag{23}$$

or, operationally,

$$v = su. \tag{24}$$

In a loss-free network under steady-state conditions, s is unitary,[6] so that

$$\begin{aligned} | s_{11} |^2 + | s_{12} |^2 &= 1, \\ | s_{22} |^2 + | s_{21} |^2 &= 1, \\ s_{11}s_{21}^* + s_{12}^*s_{22} &= 0. \end{aligned} \tag{25}$$

By algebraic manipulation of these relations (and assuming s_{22} and s_{21} to be nonvanishing), we find

$$\begin{gathered} |s_{11}|^2 - |s_{22}|^2 = |s_{21}|^2 - |s_{12}|^2, \\ \frac{|s_{11}|}{|s_{22}|} = \frac{|s_{12}|}{|s_{21}|}, \end{gathered} \tag{26}$$

and thus

$$\frac{|s_{21}|^2}{|s_{22}|^2}(|s_{21}|^2 - |s_{12}|^2) = |s_{12}|^2 - |s_{21}|^2. \tag{27}$$

The only way for this last relation to be true is for

$$|s_{12}|^2 = |s_{21}|^2. \tag{28}$$

Hence,

$$|s_{11}|^2 = |s_{22}|^2. \tag{29}$$

These two equations state that the transmission and reflection looking from one direction must equal in magnitude the transmission and reflection, respectively, looking from the other. That is, no isolator action is possible in such a loss-free network.

VI. EXPERIMENTAL BEHAVIOR OF SOME "REACTIVE" ISOLATORS

We have attempted to set up experimentally two situations which were designed to give "reactive" isolation. The first is based on the ferrite dielectric mode described by Button and Lax.[4] A rectangular waveguide partially loaded as shown in Fig. 1 was made small enough so that all the conventional TE modes were cut off. A junction was made between this section of small loaded guide with standard unloaded guide, with suitable tuning screws for matching. The reflection and transmission in both directions are plotted in Fig. 10 as a function of the transverse magnetic field. We observe the predicted "one-way" transmission but note that the loss in the reverse direction is attributable primarily to an absorption, not reflection.

A second type of reactive isolator can be made out of a field-displacement isolator[7] (Fig. 11). In this type of isolator the dominant mode can be made to have an electric field null at one face of ferrite for one direction of propagation. If we were to place a copper sheet at this point (see Fig. 11) it should not affect the propagation in this direction. However, for the reverse direction the field of this mode does not have a null. We would expect, therefore, that if the forward direction were well matched it could be made perfectly transmitting, while one might

expect a strong reflection would occur in the reverse direction, contrary to the theorem proved in Section V.*

The experiment was tried on a variant of the geometry shown in Fig. 11, using a partial height slab with the final dimensions shown in Ref. 7. In Fig. 12 the transmission and reflection in the forward and reverse directions are shown as functions of magnetic field. One can see that we can arrange just what we expected in the forward direction with a trans-

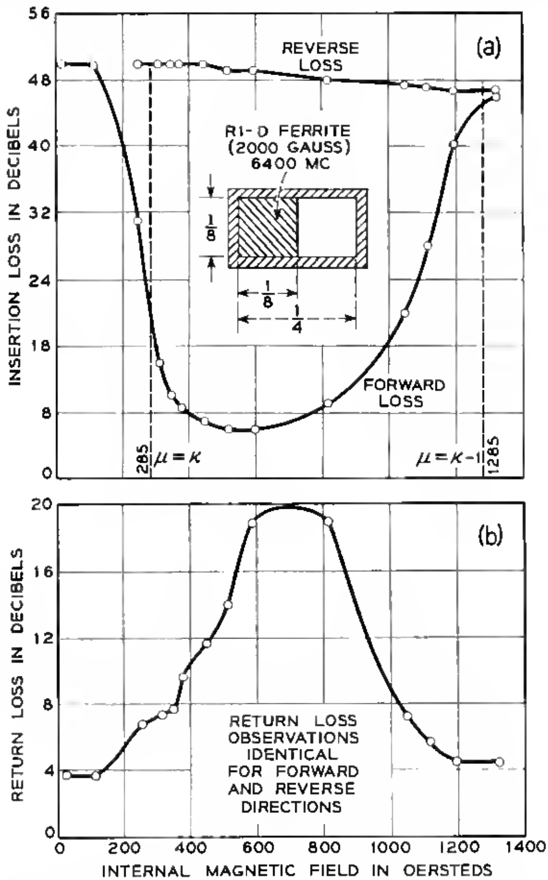

Fig. 10 — Characteristics of a ferrite-dielectric mode "reactive" isolator, showing that loss is caused by absorbtion rather than reflection: (a) insertion loss; (b) return loss.

* It may be shown that the discontinuity in H_x at the ferrite interface permits, to first order, coupling of an electric dipole to the mode having a null **E** field at the interface. This statement in itself might be viewed as a thermodynamic violation.

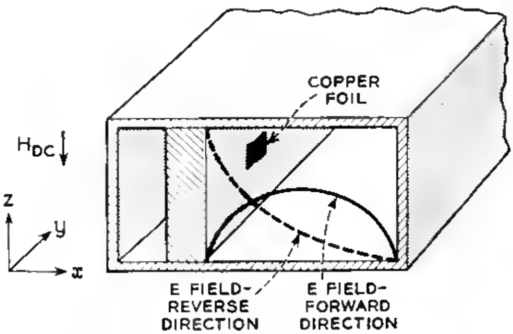

Fig. 11 — Field displacement isolator configuration with "perfectly" conducting scattering element replacing resistive sheet.

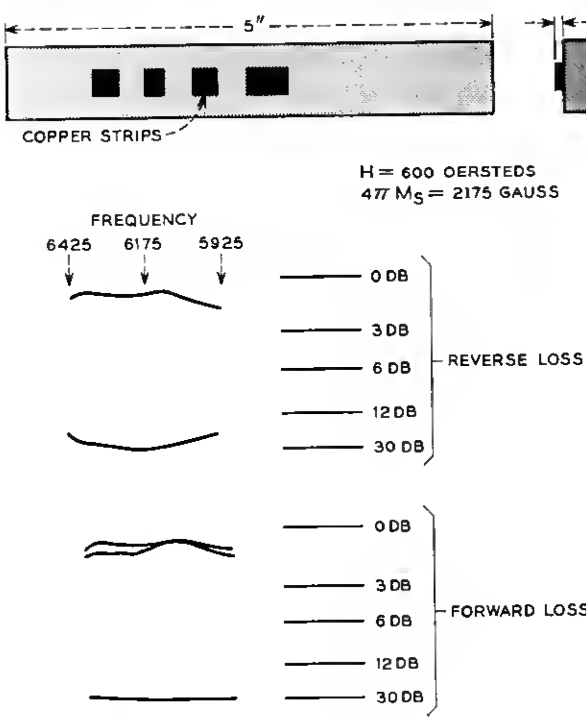

Fig. 12 — Response of copper strip insulator.

mission loss of less than 1 db. But in the reverse direction, instead of reflection, we obtain an absorption of greater than 30 db at some fields.

We thus find that when one attempts to build a theoretically impossible loss-free isolator, nature resolves the paradox, not by equalizing the transmission in both directions as predicted by the theorem in Section V, but by absorbing power in at least one direction, thus violating the assumptions of the theorem.

VII. POSSIBLE EXPLANATIONS OF "REACTIVE" ISOLATOR BEHAVIOR

It seems to us that the most reasonable explanation of the absorption that appears when a "reactive" isolator should be reflecting is caused by the excitation of gyromagnetic modes, which, as shown in Section II, tend to be lossy. It should be noticed that, for the range of magnetic fields investigated ($\mu > 0$) in Section II, there is no magnetic field for which there are propagating modes in one direction and not in the opposite direction.

Thus, it is tempting to suggest that a possible resolution of this paradox of Section VI is that, if we really had a zero loss material, we would get transmission in both directions, one set of modes carrying the power in one direction, but a different set carrying it for the opposite direction when the first set cannot propagate. For example, the FD mode in Fig. 10 could carry the power in one direction, whereas the FA modes would carry it in the other.

It can be appreciated that this suggestion requires a simple boundary to perform some rather startling feats. It must excite one mode for one direction of propagation that has one distribution of fields, say the FD mode with a uniform distribution in the z direction, while for the other direction a different mode must be excited, e.g., the FA modes with a sinusoidal variation in the z direction. In defense of the suggestion, we offer the experimental evidence described in Section IV that a simple boundary apparently can perform startling feats.

An alternate possible explanation has been offered by Walker.[8] He has proposed that power may be transmitted through cutoff modes. Thus, if the FD mode is coupled for one direction of propagation, a set of cutoff TE modes will carry the power in the reverse direction. This similarly requires some extraordinary behavior at the boundaries. In order for a cutoff mode to have appreciable amplitude at the far end, it must have an amplitude at the near end that is exponentially larger, the exponent being proportional to the length of the cutoff section. In the presence of a little loss in the material, such large amplitudes would give rise to large absorption, explaining the observed loss. Our reason for

favoring the gyromagnetic mode resolution rather than the cutoff modes is that we have experimental evidence for the coupling to the gyromagnetic modes, but probing has thus far not indicated the existence of excess fields in the vicinity of the boundary.

Another suggestion for resolving the dilemma of the Button-Lax reactive isolator was originally given us by R. L. Martin. This same suggestion is attributed independently to some work of A. D. Bressler in this matter. The viewpoint expressed was that the position of the ferrite slab should be viewed as a limiting process as the ferrite slab approaches contact with the wall. Under this situation, there is an FD mode propagating in both directions that can carry the power. Since we have considered the case mathematically in which the slab exactly contacts the wall, such a limiting procedure does not appear to have any justification. Nevertheless, if we apply this process to the TE mode equations of Lax, Button and Roth,[9] the propagation constant of the returning FD wave approaches

$$k_y \doteq \frac{1}{\delta} \operatorname{arctanh} (\kappa - \mu), \tag{30}$$

where δ is the air separation of the ferrite from the metal wall. This mode has maximum fields at the ferrite surface which fall off exponentially away from it as $e^{-k_y x}$. Thus, most of the energy in this mode is confined within a distance $1/k_y$ of the ferrite surface. As δ goes to zero, we would not be able to excite this mode from an impinging TE mode which has zero transverse field components at the only values of x where the FD mode has any amplitude. Thus, this does not appear to us a valid resolution of this paradox.

VIII. CONCLUSION

We thus see that the consideration of the gyromagnetic modes in a partially filled waveguide has led to some unusual nonreciprocal effects. Nonresonance isolation can be obtained without the deliberate introduction of loss material. Coupling to these modes tends to violate the usual symmetry arguments. Finally, they seem capable of resolving the Button-Lax "paradox" concerning reactive isolation.

We should like to acknowledge, with appreciation, the help of W. A. Dean and J. J. Kostelnick in the experimental studies.

APPENDIX A

Scattering of a TEM Mode from a Uniform Semi-Infinite Ferrite Interface Contained between Walls of Finite Resistivity

Properly, we should find the gyromagnetic modes corresponding to (6) through (9) for walls of finite conductivity. This is an extremely in-

volved process when we attempt to solve tbe partially filled waveguide problem of Section II. Since we are interested only in sbowing the breakdown of symmetry arguments for predicting coupling, in what follows we will consider a semi-infinite medium (Fig. 9) bounded by walls of finite conductivity.

We will assume no variation in the fields in the x direction ($k_x = 0$) and confine our attention only to those modes for whicb $|k_z|^2$ and $|k_y|^2$ in the ferrite are large compared to $\omega^2\epsilon_0\mu_0$. The plane wave fields in the ferrite are then given[3] by

$$E_{1F}^{\pm} = \begin{bmatrix} i\left(\frac{1-\mu}{\kappa}\right) \\ -1 \\ \pm i\mu^{-1/2} \end{bmatrix} \exp\,(\pm ik_{z_1}z - ik_y y), \tag{31}$$

$$E_{2F}^{\pm} = \begin{pmatrix} 0 \\ -1 \\ \pm i \end{pmatrix} \exp\,(\pm ik_{z_2}z - ik_y y), \tag{32}$$

$$H_{1F}^{\pm} = \frac{ik_{z_1}}{\omega\mu_0}\left(\frac{1-\mu}{\kappa\mu}\right)\begin{pmatrix} 0 \\ \pm 1 \\ i\mu^{+1/2} \end{pmatrix} \exp\,(\pm ik_{z_1}z - ik_y y), \tag{33}$$

$$H_{2F}^{\pm} = +\,\frac{i\omega\epsilon}{k_z}\begin{pmatrix} \pm i \\ \pm\dfrac{\kappa}{\mu-1} \\ \pm\dfrac{i\kappa}{\mu-1} \end{pmatrix} \exp\,(\pm ik_{z_2}z - ik_y y). \tag{34}$$

The relations corresponding to (4) and (5) are

$$k_{z_1} = -i\mu^{+1/2}k_y\,, \tag{35}$$

$$k_{z_2} = -ik_y\,. \tag{36}$$

We will take the origin of the z-axis midway between the metallic boundaries. Note that the plus and minus ($\pm$) now refer to z-directed waves, which is different from the convention used in (6) through (9).

In the metal for $\sigma \gg \omega\epsilon$, Maxwell's equations reduce to

$$\begin{aligned} \text{curl}\,\mathbf{H}_m &= \sigma\mathbf{E}_m\,, \\ \text{curl}\,\mathbf{E}_m &= i\omega\mu_0\mathbf{H}_m\,, \end{aligned} \tag{37}$$

and the wave equation [corresponding to (3)] yields the relation

$$k_y^2 + k_{zm}^2 = -i\omega\mu_0\sigma. \tag{38}$$

Let us first ask under what conditions the modes (6) through (9) are distorted by the resistive walls. This will happen when the electric fields caused by the induced current flowing in the resistive walls become comparable to the maximum electric field of the mode. Now the induced electric field is just $(1/\delta\sigma)(\mathbf{n} \times \mathbf{H}_w)$, where $\mathbf{n}$ is the surface normal, $\mathbf{H}_w$ is the magnetic field at the wall, and δ is the skin depth. Thus, (6) through (9) with $\sin\varphi = 0$ yield

$$\frac{E_1 \text{ (induced)}}{E_1 \text{ (max)}} = \frac{k_{z_1}}{\omega\mu_0\mu\sigma\delta}, \tag{39}$$

$$\frac{E_2 \text{ (induced)}}{E_2 \text{ (max)}} = \frac{1}{\sigma\delta}\frac{\omega\epsilon}{k_{z_2}}. \tag{40}$$

We see that the induced fields for mode 2 decrease for large k_z so that this mode is little affected by a finite conductivity. However, for mode 1, when $k_z \gg \omega\mu_0\mu\sigma\delta$, the induced fields are large and the mode 1 is greatly modified.

To get the correct fields for this condition we must combine the plane wave fields of (31) through (34). The problem can be simplified in this large k_z limit by noting that, for mode 1, the ratio of electric fields to magnetic fields varies as $1/k_z$, whereas for mode 2 this ratio varies as k_z. This means that, if the electric fields of the two modes are comparable, we can neglect H_2, while if the magnetic fields are comparable, we can neglect E_1. But in this latter case we have already shown that, if we retain both E_2 and H_2, we can satisfy the boundary conditions for large k_z by the unmodified mode 2 (without including H_1). Therefore we will seek the modified mode 1 from a mixture of the electric fields of mode 1 and mode 2 and neglect H_2.

This considerably simplifies our problem, since we note that $H_{1x} = 0$. By continuity across the metal-ferrite interface this also implies that $H_{mx} = 0$. Therefore, from (35) we see that in the metal we have a TE mode:

$$\mathbf{E}_m = \frac{i}{\sigma}\begin{pmatrix} k_{zm}H_{my} - k_yH_{mz} \\ 0 \\ 0 \end{pmatrix}.$$

Since div H vanishes in the metal,

$$k_{zm}H_{mz} + k_yH_{my} = 0 \tag{41}$$

and the relation (39) with the use of (38) reduces to

$$\mathbf{E}_m = \frac{\omega\mu_0}{k_{zm}} H_{my} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}. \tag{42}$$

The wave we will select in each of the two metal regions will be that which falls off exponentially away from the metal-ferrite interface. Thus, in the $+z$ region, if we take the root of k_{zm} that has a positive imaginary part, we must select the wave which varies as $e^{+ik_zm_z}$ In the $-z$ region we select e^{-ik_zmz}.

In the ferrite the total field for a particular mode (characterized by a y variation of e^{-ik_yy}) is given by

$$\begin{pmatrix} E_{1F}^{\sigma} \\ H_{1F}^{\sigma} \end{pmatrix} = A_1^{+} \begin{pmatrix} E_{1F}^{+} \\ H_{1F}^{+} \end{pmatrix} + A_1^{-} \begin{pmatrix} E_{1F}^{-} \\ H_{1F}^{-} \end{pmatrix} + A_2^{+} \begin{pmatrix} E_{2F}^{+} \\ 0 \end{pmatrix} + A_2^{-} \begin{pmatrix} E_{2F}^{-} \\ 0 \end{pmatrix}. \tag{43}$$

The boundary conditions require that E_x, E_y and H_y be continuous across the ferrite-metal interface. Let $\theta = (k_zb)/2$. At $z = +b/2$, continuity of E_x and H_y requires that

$$A_1^{+}e^{i\theta_1} + A_1^{-}e^{-i\theta_1} = \frac{k_{z1}}{\mu k_{zm}} (A_1^{+}e^{i\theta_1} - A_1^{-}e^{-i\theta_1}) \tag{44}$$

and, at $z = -b/2$,

$$A_1^{+}e^{-i\theta_1} + A_1^{-}e^{+i\theta_1} = -\frac{k_{z1}}{\mu k_{zm}} (A_1^{+}e^{-i\theta_1} - A_1^{-}e^{i\theta_1}). \tag{45}$$

For large $k_z(k_z^2 \gg \omega\mu_0\sigma)$,

$$k_{zM} = -ik_y = k_{z2} = \mu^{-1/2}k_{z1}. \tag{46}$$

With this relation, (44) and (45) have a solution

$$\tan 2\theta_1 = -\frac{2i\mu^{1/2}}{1 + \mu} \tag{47}$$

and

$$\frac{A_1^{+}}{A_1^{-}} = -1. \tag{48}$$

In order to evaluate $A_2^{\pm}$ we require E_y to vanish at $z = \pm b/2$:

$$A_1^+e^{+i\theta_1} + A_1^-e^{-i\theta_1} + A_2^+e^{+i\theta_2} + A_2^-e^{-i\theta_2} = 0, \tag{49}$$

$$A_1^+e^{-i\theta_1} + A_1^-e^{+i\theta_1} + A_2^+e^{-i\theta_2} + A_2^-e^{+i\theta_2} = 0. \tag{50}$$

These have the solution

$$A_2^+ = -A_2^- = A_1^+ \frac{\sin\theta_1}{\sin\theta_2}. \tag{51}$$

Collecting (31), (32), (33), (43), (48) and (50), we obtain the resultant fields in the ferrite:

$$\mathbf{E}_{1f}^{\sigma} = F_1 \begin{bmatrix} -i\dfrac{1-\mu}{\kappa}\sin k_{z1}z \\[2ex] \sin k_{z1}z - \dfrac{\sin k_{z1}\dfrac{b}{2}}{\sin k_{z2}\dfrac{b}{2}}\sin k_{z2}z \\[2ex] \mu^{-1/2}\cos k_{z1}z - \dfrac{\sin k_{z1}\dfrac{b}{2}}{\sin k_{z2}\dfrac{b}{2}}\cos k_{z2}z \end{bmatrix} e^{-ik_yy}, \tag{52}$$

$$\mathbf{H}_{1f}^{\sigma} = \frac{k_{z1}}{\omega\mu_0\mu}\,\frac{1-\mu}{\kappa}\,F_1\begin{pmatrix} 0 \\ -\cos k_{z1}z \\ \mu^{1/2}\sin k_{z1}z \end{pmatrix} e^{-ik_yy}, \tag{53}$$

where

$$k_{z1} = \frac{m\pi}{2b} + \frac{i}{2b}\operatorname{arctanh}\frac{2\mu^{1/2}}{1+\mu} \qquad m = 1, 2, \cdots, \tag{54}$$

$$k_{z2} = \mu^{-1/2}k_{z1} = ik_y, \tag{55}$$

and the z-axis is now considered to have its origin halfway between the metal walls.

If the normal modes of the finite conductivity guide are enumerated, $\mathbf{E}_j$ and $\mathbf{H}_j$, we can expand the incident wave, $(\mathbf{E}/\mathbf{H})$, in terms of them according to the relation

$$\begin{pmatrix}\mathbf{E}\\ \mathbf{H}\end{pmatrix} = \sum_j \begin{pmatrix}\mathbf{E}_j\\ \mathbf{H}_j\end{pmatrix} = \sum A_j\begin{pmatrix}\mathbf{e}_j\\ \mathbf{h}_j\end{pmatrix}, \tag{56}$$

where the amplitude A_j is given in terms of the modes of the adjoint set $\mathbf{e}_j^{\dagger}$ and $\mathbf{h}_j^{\dagger}$, (Appendix B) by

$$\int_S (\mathbf{E} \times \mathbf{h}_j^\dagger + \mathbf{e}_j^\dagger \times \mathbf{H}) \cdot \mathbf{dS}, \tag{57}$$

and the lower case vectors $\mathbf{e}_j$, $\mathbf{h}_j$, represent a normalized set of modes, which obey the normalization condition:

$$1 = \int_S (\mathbf{e}_j \times \mathbf{h}_j^\dagger + \mathbf{e}_j^\dagger \times \mathbf{h}_j) \cdot dS. \tag{58}$$

From (27) it can be seen that A_j is nonvanishing for the modes $E_{1f}{}^\sigma$ of large wave number [(22) and (23)], when the TE mode ($\mathbf{E}/\mathbf{H}$) is incident; i.e., these modes will be excited even though there is no variation of the boundary in the direction of the magnetic field to excite them.

It is of interest to evaluate the total amount of power contained in these higher-order modes. By our simplification of the problem to a semi-infinite plane, all of the higher-order modes are cut off [equation (25)]. In order to calculate a power flow, therefore, we need to introduce a little loss into the ferrite medium. We do this by allowing the components, μ and k, of the permeability tensor to take on the complex values, $\mu' - i\mu''$ and $\kappa' - i\kappa''$ and the dielectric constant, ϵ, to become $\epsilon' - i\epsilon''$.

The total power across a cross section, S_y, is given by

$$P = \frac{1}{2} \sum_{j,l} \int (\mathbf{E}_j \times \mathbf{H}_l^* + \mathbf{E}_l^* \times \mathbf{H}_j) \cdot \mathbf{dS}_y = \sum_{j,l} P_{jl}. \tag{59}$$

From Maxwells' equations (1) one can obtain the identity

$$\begin{aligned} \operatorname{div} (\mathbf{E}_j \times \mathbf{H}_l^* + \mathbf{E}_l^* \times \mathbf{H}_j) &= i\omega[\mu_0(\mathbf{H}_j \cdot \mathbf{T}^* \cdot \mathbf{H}_l^* - \mathbf{H}_l^* \cdot \mathbf{T} \cdot \mathbf{H}_j) \\ &\qquad + (\mathbf{E}_j \cdot \epsilon^* \mathbf{E}_l - \mathbf{E}_l^* \cdot \epsilon \mathbf{E}_y)] \\ &= 2\omega[\mu_0 \mathbf{H}_j \cdot \tau \cdot \mathbf{H}_l^* + \epsilon'' \mathbf{E}_j \cdot \mathbf{E}_l], \end{aligned} \tag{60}$$

where τ is the tensor $(1/2i)(T^* - T^T)$ and is given by

$$\tau = \begin{pmatrix} \mu'' & i\kappa'' & 0 \\ -i\kappa'' & \mu'' & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{61}$$

The application of Gauss' theorem to (30) in a small-volume element bounded by $S_y(y)$, and $S_y(y + dy)$ yields

$$\begin{aligned} \frac{\partial}{\partial y} \int & (\mathbf{E}_j \times \mathbf{H}_l^* + \mathbf{E}_l^* \times \mathbf{H}_j) \cdot \mathbf{dS}_y \\ &= 2\omega \int (\mu_0 \mathbf{H}_j \cdot \tau \cdot \mathbf{H}_l^* + \epsilon'' \mathbf{E}_j \cdot \mathbf{E}_l^*) dS_y, \end{aligned} \tag{62}$$

whereupon, from (29),

$$P_{jl} = \frac{i\omega \int (\mu_0 \mathbf{H}_j \cdot \tau \cdot \mathbf{H}_l^* + \epsilon'' \ \mathbf{E}_j \cdot \mathbf{E}_l^*) dS_y}{k_{y_j} - k_{y_l}^*}$$
$$= \frac{i\omega A_j A_j^* \int (\mu_0 \mathbf{h}_j \cdot \tau \cdot \mathbf{h}_l^* + \epsilon'' \mathbf{e}_j \cdot \mathbf{e}_l^*) dS_y}{k_{y_j} - k_{y_l}^*}. \tag{63}$$

Let us examine just a part of the power expended in the diagonal power components, P_{jj}, for the resistive wall modes, (22) and (23), which we will call P_{mm}. Here we will be interested in the dependence on m for large m to normalize (22) and (23) according to (28). The amplitudes F_1 for the normal modes must vary as

$$F_1 F_1^{\dagger} \sim m. \tag{64}$$

If we evaluate $A_j A_j^*$ for the TEM mode as modified by the ferrite and resistive wall (as discussed earlier) we find

$$A_j A_j^* \sim \frac{E_x^2}{m}.$$

Finally, the component of power, P_{mm}, in the mth mode varies as

$$P_{mm} \sim \frac{\mu'' \mu^{1/2}}{m} E_x^2 \sim \frac{\mu''}{m\sigma^2} H_y^2. \tag{66}$$

If we sum over all the modes, m, we find that the total power contained in these higher-order modes has a logarithmic singularity. This means that the reflection and transmission coefficients at the boundary must have been such as to reduce the amplitudes of the higher-m modes. This is true no matter how high the conductivity becomes. Thus, a reasonable interpretation of this divergence is that there must be a finite coupling to the gyromagnetic modes, even those of lower m, at the boundary. This argument leads to the suggestion that, if we have the proper explanation of the observed coupling, this coupling might become independent of wall conductivity at high enough conductivities.

APPENDIX B

Orthogonality Relationships

Orthogonality relations in generalized media are well covered in the literature.† Nevertheless, to show explicit forms. we include derivations

† See, typically, Refs. 10 and 11.

specifically directed to gyromagnetic media of the form discussed in this paper.

A real medium supporting electromagnetic propagation is characterized by free space parameters, μ_0 and ϵ_0, and by relative electric and magnetic susceptibilities. Given only a magnetic anisotropy produced by an applied magnetic field in gyromagnetic media, we further characterize the medium by the Polder tensor T, as well as by the ordinary scalar relative dielectric constant, ϵ.

The Polder tensor contains components which are all complex because of losses. Nevertheless, the transverse off-diagonal terms of this tensor are perfectly skew, and the sign associated with either component is prescribed by the direction of precession. The transpose of this tensor simply reverses the signs of the off-diagonal components and corresponds to time reversal in the dynamic classical equation of the spin.

Let us define a medium reciprocal to the real medium of the guide such that the following transformations hold:

$$\mu_0 \to -\mu_0 ,$$

$$\epsilon_0 \to -\epsilon_0 ,$$

$$T \to T',$$

where T' is an operator yet to be defined. The reciprocal fields are $\mathbf{E}'$ and $\mathbf{H}'$ and satisfy the Maxwell equations

$$\text{curl } \mathbf{H}' = -i\omega\epsilon_0\epsilon\mathbf{E}', \tag{67}$$

$$\text{curl } \mathbf{E}' = i\omega\mu_0 T' \cdot \mathbf{H}'. \tag{68}$$

We have the identity

$$\begin{aligned} &\text{div } (\mathbf{E} \times \mathbf{H}' + \mathbf{E}' \times \mathbf{H}) \\ &\quad = (\mathbf{E} \cdot \text{curl } \mathbf{H}' + \mathbf{E}' \cdot \text{curl } \mathbf{H}) - (\mathbf{H} \cdot \text{curl } \mathbf{E}' + \mathbf{H}' \cdot \text{curl } \mathbf{E}). \end{aligned} \tag{69}$$

From (67) and (68), the right-hand side of (69) becomes

$$i\omega[\epsilon_0\epsilon(-\mathbf{E} \cdot \mathbf{E}' + \mathbf{E}' \cdot \mathbf{E}) + \mu_0(\mathbf{H}' \cdot T \cdot \mathbf{H} - \mathbf{H} \cdot T' \cdot \mathbf{H}')].$$

If T' is defined such that $T' = T^T$, the right-hand side of (69) vanishes identically, and

$$\text{div } (\mathbf{E} \times \mathbf{H}' + \mathbf{E}' \times \mathbf{H}) = 0. \tag{70}$$

The reciprocal system bears the relation to the real system of creating solutions identical to those of the real system but having a negative time variation. If, then, there exists a solution of form $\epsilon^{i(\beta y + \omega t)}$ in the real

system there exists solutions

$$\epsilon^{i[\beta y+\omega(-t)]} = \epsilon^{-i[(-\beta)y+\omega t]}$$

in the reciprocal system, implying, for the reciprocal propagation constant β',

$$\beta' = -\beta. \tag{71}$$

We now perform an integration of (70) over a differential volume of a cylindrical waveguide formed by two infinitesimally separated planes normal to the guide axis along the y direction, and intersecting the guide walls. From Gauss' theorem,

$$\int (\mathbf{E} \times \mathbf{H}' + \mathbf{E}' \times \mathbf{H}) \cdot d\mathbf{S} = 0. \tag{72}$$

Since $\mathbf{E} \times d\mathbf{S}$ vanishes on the guide wall, the surface integral takes on value only over the two transverse planes normal to the axis. The left-hand side of (72) has a value equal to the difference of the surface integrals over these adjacent planes, viz.:

$$dy \frac{\partial}{\partial y} \int (\mathbf{E} \times \mathbf{H}' + \mathbf{E}' \times \mathbf{H}) \cdot d\mathbf{A} = 0, \tag{73}$$

where $\mathbf{A}$ is the transverse cross section of the guide.

Let us assume a mode of order k for $\mathbf{E}$ and order j for $\mathbf{E}'$:

$$\begin{aligned} \mathbf{E} &= \mathbf{E}_k(x,y)\epsilon^{-i\beta_k y} \\ \mathbf{E}' &= \mathbf{E}^{(j)}(x,y)\epsilon^{-i\beta'^{(j)}y}. \end{aligned}$$

Then, from (73),

$$(\beta_k - \beta_j) \int [\mathbf{E}_k \times \mathbf{H}^{(j)} + \mathbf{E}^{(j)} \times \mathbf{H}_k] \cdot d\mathbf{A} = 0, \tag{74}$$

where we have employed (71) to transform $\beta'^{(j)}$ to $-\beta_j$. Equation (74) provides the final result:

$$\frac{\int [\mathbf{E}_k \times \mathbf{H}^{(j)} + \mathbf{E}^{(j)} \times \mathbf{H}_k] \cdot d\mathbf{A}}{\int [\mathbf{E}_k \times \mathbf{H}^{(k)} + \mathbf{E}^{(k)} \times \mathbf{H}_k] \cdot d\mathbf{A}} = \delta_{jk}. \tag{75}$$

The notational change to that employed in (57) is evident.

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